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Variance-Constrained Distributed Filtering for Time-varying Systems with Multiplicative Noises and Deception Attacks over Sensor Networks

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***Abstract*—This paper is concerned with the variance-constrained distributed filtering problem for a class of time-varying systems subject to multiplicative noises, unknown but bounded disturbances and deception attacks over sensor networks. The available measurements at each sensing node are collected not only from the individual sensor but also from its neighbors according to the given topology. A new deception attack model is proposed where the malicious signals are injected by the adversary into both control and measurement data during the process of information transmission via the communication network. By resorting to the recursive linear matrix inequality approach, a sufficient condition is established for the existence of the desired filter satisfying the prespecified requirements on the estimation error variance. Subsequently, an optimization problem is formulated in order to seek the filter parameters ensuring the locally optimal filtering performance at each time instant. Finally, an illustrative example is presented to demonstrate the effectiveness and applicability of the proposed algorithm.**

***Index Terms*—Distributed filtering, multiplicative noises, deception attacks, variance constraints, sensor networks, unknown but bounded disturbances**

I. Introduction

The rapid development of microelectronic technologies over the past few decades has boosted the utilization of networks which consist of a large number of devices implemented distributively for sensing, communication as well as actuating [28]. A typical example is the sensor network which has been found wide applications ranging from various industrial branches to critical infrastructures such as military facilities and power grids, see [3], [23] and the references therein. In particular, the state estimation or filtering problems over sensor networks have posed several emerging challenges, which have attracted an ever-increasing research attention within the signal processing and control community.

So far, considerable effort has been devoted to the investigation of the distributed filtering problems and a number of strategies have been developed based on the Kalman filtering theory or the H_∞ filtering theory, see [5], [6], [13], [14], [16], [26], [31] for some recent results. As is well known, the Kalman filtering technique requires an assumption of Gaussian distributions for the process and measurement noises, while the H_∞ theory can be utilized in the occasion when the disturbances are assumed to have bounded energy. However, in many real-world engineering practice, due to a variety of reasons (e.g., man-made electromagnetic interference), it is much more appropriate to model the disturbances/noises as signals that are unknown but bounded in certain sets rather than Gaussian noises or energy-bounded disturbances [8], [12], [33]. Obviously, in such a case, the aforementioned conventional techniques based on Kalman filtering or H_∞ filtering frameworks are no longer effective. Consequently, the filtering problems for systems subject to the so-called unknown but bounded noises have exerted tremendous fascination on researchers as well as engineers within the signal processing community. So far, quite a few methodologies have been exploited, see e.g. [9], [24]. Nevertheless, in the general context of *sensor networks*, little progress has been made on the corresponding filtering problems owing probably to the difficulty in quantifying the filtering performance with respect to the unknown but bounded noises as well as the complexity which stems from the coupling between communication topology and the system dynamics.

Along with the pervasive utilization of open yet unprotected communication networks, the sensor networks are vulnerable to cyber threat [10]. As a result, the security of network, which is of utmost importance in the networked-related systems, has provoked an increasing research interest and a multitude of results have been reported in the literature, see [29] and the references therein. In general, there are mainly two types of cyber attacks which can affect the systems behavior directly or through feedback, namely, the denial-of-service (DoS) attacks [21] and the deception attacks [7]. Different from the DoS attack which deteriorates the system performance by preventing the information from reaching the destination, the deception attack aims at manipulating the system toward the adversaries' desired behaviors by injecting deception information to the control actions or system measurements. A quintessen-

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tial example of deception attack should be cited is that in the context of target tracking, the electronic countermeasure (ECM) techniques are always developed to deceive radars. By manipulating and rebroadcasting the Doppler information of the target, the deception signals can be injected and maintained through frequency-shifted copies of the radar's signals, thereby degrading the tracking performance [30].

To tackle the filtering/control problems for the systems under cyber attacks, several approaches have been developed, including linear programming [29], linear matrix inequality method [18], [27], game theory approach [38], to name but a few key ones. However, when it comes to the distributed filtering issues over sensor networks, the corresponding results have been scattered, although some interesting initial results have appeared, see e.g. [32], [34]. To the best of the authors' knowledge, up to now, the research on distributed filtering is far from adequate when the communication networks are affected by attacks. The difficulty probably lies in the lack of appropriate attack models which, on one hand, could comprehensively reflect the engineering practice, and on the other, can be handled systematically within the existing frameworks. There are still a number of open yet challenging problems deserving further investigation.

In response to the above discussion, it is our objective in this paper to design a distributed filter for the discrete time-varying systems with multiplicative noises, unknown but bounded disturbances and deception attacks such that the estimation error variance of each sensing node is constrained by a prespecified upper bound at each time instant. Note that the specific time-varying nature of the addressed system imposes substantial challenges on both performance analysis and filter design, not to mention the difficulties stemming from the coupling between the communication topology and the deception attacks, especially when the error variances are required to satisfy certain upper bounds at each time step. Therefore, we shall make the first of the few attempts to develop new paradigms to solve the so-called variance-constrained distributed filtering problem subject to deception attacks over sensor networks.

The main contributions can be highlighted as follows: (i) a unified framework is established within which the variance-constrained distributed filtering problem can be conveniently handled in the presence of multiplicative noises, unknown but bounded disturbances and deception attacks; (ii) the proposed deception attack model is novel, which provides a better way to reflect the engineering reality in a comprehensive way by simultaneous consideration of several network-induced complexities; and (iii) a sufficient condition is proposed to recursively determine the filter parameters capable of guaranteeing the prespecified upper bound on the estimation error variances at each time instant.

The rest of this paper is organized as follows: Section II formulates the variance-constrained distributed filter design problem for the discrete time-varying system subject to multiplicative noises, unknown but bounded disturbances and deception attacks. The main results are presented in Section III where a sufficient condition for the existence of the desired filter is given in terms of recursive linear matrix inequalities. Section IV gives a numerical example. Section V is our conclusion.

Notation The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\mathbf{1}_n$ denotes an n-dimensional column vector with all ones. I_n denotes the identity matrix of n dimensions. The notation $X \geq Y$ (respectively $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively positive definite). The superscript "T" denotes the transpose. Z^+ stands for all the positive integers. $\mathrm{diag}\{F_1, F_2, \ldots\}$ denotes a block diagonal matrix whose diagonal blocks are given by F_1, F_2 .... The notation $\mathrm{diag}_n\{A_i\}$ represents the block diagonal matrix $\mathrm{diag}\{A_1, A_2, \ldots, A_n\}$. The notation $\mathrm{col}_n\{x_i\}$ denotes the column vector $[x_1^{\mathrm{T}} \;\; x_2^{\mathrm{T}} \;\; \ldots \;\; x_n^{\mathrm{T}}]^{\mathrm{T}}$. For matrices $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{p\times q}$, their Kronecker product is a matrix in $\mathbb{R}^{mp\times nq}$ denoted as $A \otimes B$. $\|a\|_2^2$ where a is a vector represents $a^{\mathrm{T}}a$, while $\|a\|_A^2$ means $a^{\mathrm{T}}Aa$. $\mathrm{tr}[A]$ means the trace of matrix A.

II. Problem Formulation

In this paper, it is assumed that the sensor network has N sensor nodes which are distributed in the space according to a specific interconnection topology characterized by a directed graph $\mathscr{G} = (\mathscr{V}, \mathscr{M}, \mathscr{L})$, where $\mathscr{V} = \{1, 2, ..., N\}$ denotes the set of sensor nodes, $\mathscr{M} \subseteq \mathscr{V} \times \mathscr{V}$ is the set of edges, and $\mathscr{L} = [\theta_{ij}]_{N\times N}$ is the nonnegative adjacency matrix associated with the edges of the graph, i.e., $\theta_{ij} > 0 \Leftrightarrow \mathrm{edge}\,(i,j) \in \mathscr{M}$, which means that there is information transmission from sensor j to sensor i. If $(i,j) \in \mathscr{M}$, then node j is called one of the neighbors of node i. Also, we assume that $\theta_{ii} = 1$ for all $i \in \mathscr{V}$, and therefore, (i,i) can be regarded as an additional edge. The set of neighbors of node $i \in \mathscr{V}$ plus the node itself is denoted by $\mathscr{N}_i \triangleq \{j \in \mathscr{V} | (i,j) \in \mathscr{M}\}$.

A. System model

Consider a discrete time-varying system described by

$$x_{k+1} = \Big(A_k + \sum_{l=1}^{q} \alpha_{l,k} A_{l,k}\Big)x_k + B_k u_k + D_k w_k, \qquad (1)$$

with the measurements from N sensors given by

$$y_{i,k} = C_{i,k}x_k + E_{i,k}v_k, \quad i = 1, 2, \ldots, N \qquad (2)$$

where $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^m$ and $y_{i,k} \in \mathbb{R}^p$ are, respectively, the state, the known input and the measurement output of sensor i; $\alpha_{l,k} \in \mathbb{R}$ $(l = 1, 2, \ldots, q)$ are sequences of uncorrelated zero-mean Gaussian noises with unitary covariances; $w_k \in \mathbb{R}^\omega$ and $v_k \in \mathbb{R}^\nu$ represent the unknown but bounded process and measurement disturbances, respectively; A_k, B_k, D_k, $C_{i,k}$ and $E_{i,k}$ are real-valued time-varying matrices of compatible dimensions. The following definitions as well as assumptions are needed for the further development.

Definition 1: Let Ψ_1 and Ψ_2 be some real matrices with $\Psi \triangleq \Psi_2 - \Psi_1 > 0$. A nonlinearity $\varphi(\cdot)$: $\mathbb{R}^n \mapsto \mathbb{R}^n$ is said to satisfy the sector condition with respect to Ψ_1 and Ψ_2 if

$$\big(\varphi(\varepsilon) - \Psi_1\varepsilon\big)^{\mathrm{T}}\big(\varphi(\varepsilon) - \Psi_2\varepsilon\big) \leq 0, \quad \forall \varepsilon \in \mathbb{R}^n. \qquad (3)$$

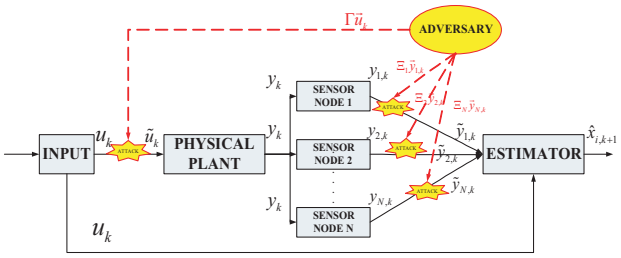

Fig. 1. Deception attack model.

In this case, the sector-bounded nonlinearity $\varphi(\cdot)$ is said to belong to the sector $[\Psi_1, \Psi_2]$.

Definition 2: A bounded ellipsoid $\mathscr{E}(c, P, n)$ of $\mathbb{R}^n$ with a nonempty interior can be defined by

$$\mathscr{E}(c, P, n) \triangleq \{x \in \mathbb{R}^n : (x-c)^{\mathrm{T}} P^{-1}(x-c) \leq 1\} \tag{4}$$

where $c \in \mathbb{R}^n$ is the center of $\mathscr{E}(c, P, n)$ and $P > 0$ is a positive definite matrix that specifies the ellipsoid's shape and orientation.

Assumption 1: The unknown but bounded noises w_k and v_k are confined to the following specified ellipsoids:

$$\begin{aligned} w_k \in \mathscr{E}(0, W_k, \omega) \triangleq & \{w_k \in \mathbb{R}^\omega : w_k^{\mathrm{T}} W_k^{-1} w_k \leq 1\} \\ v_k \in \mathscr{E}(0, V_k, \nu) \triangleq & \{v_k \in \mathbb{R}^\nu : v_k^{\mathrm{T}} V_k^{-1} v_k \leq 1\} \end{aligned} \tag{5}$$

where $W_k > 0$ and $V_k > 0$ are known positive definite matrices of appropriate dimensions.

B. Deception attack model

In this paper, we investigate the following deception attack scenario. Attempting to deteriorate the filtering performance, the adversary injects certain deception signals into the true signals of the control input u_k and the measurement outputs $y_{i,k}$ during the process of data transmission through the communication networks. Such an attack scenario can be illustrated by Fig. 1.

Before giving the deception attack model, we make some further assumptions on the system knowledge that are possessed by the adversary for implementing a successful attack. In this paper, it is assumed that the adversary has sufficient resources and adequate knowledge to arrange a successful attack [25]. Specifically, the adversary, in the first place, knows the accurate values of the control input u_k and the measurement output $y_{i,k}$ in real time, and in the second place, has the ability to modify the true values of u_k and $y_{i,k}$ to arbitrary ones. Moreover, the attacks are arranged in a coordinated fashion where the deception signals are injected into each communication channel simultaneously to maximize the impact to the plant/estimator [29].

The signals used by the adversary for the deception attacks are generated as follows:

$$\begin{cases} \vec{u}_k = -u_k + \delta_k \\ \vec{y}_{i,k} = -y_{i,k} + \vartheta_{i,k}, \quad i = 1, 2, \ldots, N \end{cases} \tag{6}$$

where δ_k and $\vartheta_{i,k}$ $(i = 1, 2, \ldots, N)$ are the unknown but bounded signals belonging to the following ellipsoids:

$$\begin{aligned} \delta_k \in \mathscr{E}(0, S_k, m) \triangleq & \{\delta_k \in \mathbb{R}^m : \delta_k^{\mathrm{T}} S_k^{-1} \delta_k \leq 1\}, \\ \vartheta_{i,k} \in \mathscr{E}(0, R_{i,k}, p) \triangleq & \{\vartheta_{i,k} \in \mathbb{R}^p : \vartheta_{i,k}^{\mathrm{T}} R_{i,k}^{-1} \vartheta_{i,k} \leq 1\} \end{aligned} \tag{7}$$

with S_k and $R_{i,k}$ $(i = 1, 2, \ldots, N)$ being positive definite matrices of compatible dimensions.

Remark 1: In (7), δ_k and $\vartheta_{i,k}$ $(i = 1, 2, \ldots, N)$, which have been assumed to be unknown but confined to certain ellipsoidal sets, are used by the adversary to generate the deception attack signals. It should be noted that δ_k and $\vartheta_{i,k}$ have similar forms with the process noise w_k and the measurement noise v_k, and are therefore difficult to be distinguished by the detectors. On the other hand, the most widely implemented attack detector in the practical applications, namely, the χ^2 detector, is only effective when the noises obey Gaussian distribution [1]. As such, the utilization of unknown but bounded signals could help to pass through the χ^2 detector. In other words, from the adversary's perspective, it is practically reasonable to constrain the malicious signals δ_k and $\vartheta_{i,k}$ $(i = 1, 2, \ldots, N)$ within given ellipsoidal sets.

Remark 2: In engineering practice, attack detectors are categorized as a software barrier, and there are some other "hard" physical constraints that the adversary would need to face. Such physical constraints include device saturations, bandwidth limitations, channel fading and signal quantizations [6]. The kinds of hardware constraints should be taken into consideration if we are to establish a comprehensive yet realistic deception attack model. On the other hand, such constraints inevitably bring in new challenges that demand new techniques in analyzing the performance and design the filters.

Based on the discussions in Remark 2 and from Fig. 1, we can reformulate the actual control input $\tilde{u}_k$ (sent to the plant) and the actual measurement outputs $\tilde{y}_{i,k}$ (fed to the estimator) by

$$\begin{cases} \tilde{u}_k = u_k + \Gamma \vec{u}_k \\ \tilde{y}_{i,k} = y_{i,k} + \Xi_i \vec{y}_{i,k}, \quad i = 1, 2, \ldots, N \end{cases} \tag{8}$$

where the matrices Γ and Ξ_i represent the physical constraints imposed on the attack signals and are assumed to be of the following forms:

$$\begin{cases} \Gamma = \mathrm{diag}\{\gamma_1, \gamma_2, \ldots, \gamma_m\}, \\ \Xi_i = \mathrm{diag}\{\xi_{i,1}, \xi_{i,2}, \ldots, \xi_{i,p}\}, \quad i = 1, 2, \ldots, N. \end{cases} \tag{9}$$

Here, the entries of Γ and Ξ_i have upper- and lower-bounds that are expressed as follows:

$$\begin{cases} 0 \leq \underline{\gamma}_j \leq \gamma_j \leq \bar{\gamma}_j < \infty, \quad j = 1, 2, \ldots, m \\ 0 \leq \underline{\xi}_{i,s} \leq \xi_{i,s} \leq \bar{\xi}_{i,s} < \infty, \ s = 1, 2, \ldots, p \end{cases} \tag{10}$$

where $0 \leq \underline{\gamma}_j < 1$ and $\bar{\gamma}_j \geq 1$ are known scalars representing the lower- and upper-bounds on γ_j, and $0 \leq \underline{\xi}_{i,s} < 1$ and $\bar{\xi}_{i,s} \geq 1$ are known scalars describing the lower- and upper-bounds on $\xi_{i,s}$, respectively.

Remark 3: We now take the matrix Γ as an example to illustrate how the upper- and lower-bounds on γ_j affect the behavior of the deception attack signal $\vec{u}_k$ (the impact on

$\vec{y}_{i,k}$ from $\xi_{i,s}$ can be analyzed similarly). Specifically, when $\gamma_j = 1$, it means that the jth entry of the deception signal $\vec{u}_k$ can be injected correctly into the corresponding true u_k as the adversary plans, otherwise $\vec{u}_k$ might be unexpectedly (from the attacker's perspective) degraded ($0 \leq \gamma_j < 1$) or amplified ($\gamma_j > 1$). In this sense, the model (8)–(10) offers a comprehensive and realistic means to reflect the influence on the attacks resulting from the physical constraints as well as the network-induced complexities.

By denoting

$$\underline{\Gamma} \triangleq \text{diag}\{\underline{\gamma}_1, \underline{\gamma}_2, \ldots, \underline{\gamma}_m\}, \quad \bar{\Gamma} \triangleq \text{diag}\{\bar{\gamma}_1, \bar{\gamma}_2, \ldots, \bar{\gamma}_m\},$$
$$\underline{\Xi}_i \triangleq \text{diag}\{\underline{\xi}_{i,1}, \underline{\xi}_{i,2}, \ldots, \underline{\xi}_{i,p}\}, \ \bar{\Xi}_i \triangleq \text{diag}\{\bar{\xi}_{i,1}, \bar{\xi}_{i,2}, \ldots, \bar{\xi}_{i,p}\},$$

we can rewrite (10) into the following compact forms:

$$\begin{cases} \underline{\Gamma} \leq \Gamma \leq \bar{\Gamma}, \\ \underline{\Xi}_i \leq \Xi_i \leq \bar{\Xi}_i, \quad i = 1, 2, \ldots, N. \end{cases} \tag{11}$$

In what follows, for the convenience of later derivation, we divide the deception attack signals $\Gamma\vec{u}_k$ and $\Xi_i\vec{y}_{i,k}$ as follows:

$$\begin{cases} \Gamma\vec{u}_k = \underline{\Gamma}\vec{u}_k + \varphi(\vec{u}_k), \\ \Xi_i\vec{y}_{i,k} = \underline{\Xi}_i\vec{y}_{i,k} + \psi_i(\vec{y}_{i,k}), \quad i = 1, 2, \ldots, N. \end{cases} \tag{12}$$

It then can be easily checked that

$$\begin{cases} \varphi^{\mathrm{T}}(\vec{u}_k)\big(\varphi(\vec{u}_k) - \tilde{\Gamma}\vec{u}_k\big) \leq 0, \\ \psi_i^{\mathrm{T}}(\vec{y}_{i,k})\big(\psi_i(\vec{y}_{i,k}) - \tilde{\Xi}_i\vec{y}_{i,k}\big) \leq 0, \quad i = 1, 2, \ldots, N \end{cases} \tag{13}$$

where $\tilde{\Gamma} \triangleq \bar{\Gamma} - \underline{\Gamma} > 0$ and $\tilde{\Xi}_i \triangleq \bar{\Xi}_i - \underline{\Xi}_i > 0$ are positive definite matrices. Clearly, it follows from Definition 1 that $\varphi(\vec{u}_k)$ and $\psi_i(\vec{y}_{i,k})$ are vector-valued nonlinear functions satisfying the sector condition and belonging to the sectors $[0, \tilde{\Gamma}]$ and $[0, \tilde{\Xi}_i]$ $(i = 1, 2, \ldots, N)$, respectively.

C. Design objective

On account of the deception attacks discussed above, the original system (1)–(2) should be reformulated by

$$\begin{cases} x_{k+1} = \left(A_k + \sum_{l=1}^{q} \alpha_{l,k} A_{l,k}\right) x_k + B_k\tilde{u}_k + D_k w_k, \\ \tilde{u}_k = u_k + \Gamma\vec{u}_k, \\ \tilde{y}_{i,k} = y_{i,k} + \Xi_i\vec{y}_{i,k}, \quad i = 1, 2, \ldots, N. \end{cases} \tag{14}$$

For the system (14), at each sensing node i ($i = 1, 2, \ldots, N$), the following filter structure is adopted:

$$\hat{x}_{i,k+1} = G_{i,k}\hat{x}_{i,k} + B_k u_k + \sum_{j \in \mathcal{N}_i} \theta_{ij} K_{ij,k}(\tilde{y}_{i,k} - C_{i,k}\hat{x}_{i,k}) \tag{15}$$

where $\hat{x}_{i,k} \in \mathbb{R}^n$ is the estimate of the state x_k based on the ith sensing node, and the matrices $G_{i,k}$ and $K_{ij,k}$ $(i, j = 1, 2, \ldots, N)$ are the filter parameters to be determined.

Assumption 2: The initial state x_0 of the system (14) and its estimate values from each sensing node, namely, $\hat{x}_{i,0}$ $(i = 1, 2, \ldots, N)$, satisfy:

$$\mathbb{E}\left\{(x_0 - \hat{x}_{i,0})(x_0 - \hat{x}_{i,0})^{\mathrm{T}}\right\} \leq \Phi_0 \tag{16}$$

where $\Phi_0 > 0$ is a known positive definite matrix.

The distributed filtering problem under investigation is to estimate the state of the system (1) using a network of filters connected according to the graph $\mathscr{G}$ with the guarantee of variance constraints on the estimation errors. Specifically, the objective of this paper is twofold. For the system (1)–(2) subject to the deception attacks (6), let the communication graph $\mathscr{G}$ and the sequence of positive definite matrices $\{\Phi_k\}_{k\geq 0}$ (prespecified constraints on the estimation error variance) be given. It is our first aim to design the sequences of filtering parameters $\{G_{i,k}\}_{k\geq 0}$ and $\{K_{ij,k}\}_{k\geq 0}$ in (15) subject to the given couple $(\mathscr{G}, \{\Phi_k\}_{k\geq 0})$ such that the following inequalities are satisfied for all $k \geq 0$:

$$\mathbb{E}\left\{(x_k - \hat{x}_{i,k})(x_k - \hat{x}_{i,k})^{\mathrm{T}}\right\} \leq \Phi_k, \quad i = 1, 2, \ldots, N. \tag{17}$$

Secondly, within the proposed framework, an optimization problem will be considered for minimizing Φ_k in the sense of matrix trace at each time instant to ensure the locally optimal filtering performance. This problem will be referred to as a variance-constrained distributed filtering problem subject to deception attacks.

III. Distributed Filter Design

In this section, we will design a distributed filter of form (15) for system (1)–(2) subject to multiplicative noises, unknown but bounded disturbances and deception attacks. A sufficient condition for the existence of the desired filter will be formulated in terms of a set of recursive linear matrix inequalities (RLMIs). First, two lemmas which are useful for our subsequent development are introduced as follows.

Lemma 1: (Schur Complement Lemma) Given constant matrices $\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3$ where $\mathcal{S}_1 = \mathcal{S}_1^{\mathrm{T}}$ and $0 < \mathcal{S}_2 = \mathcal{S}_2^{\mathrm{T}}$, then $\mathcal{S}_1 + \mathcal{S}_3^{\mathrm{T}}\mathcal{S}_2^{-1}\mathcal{S}_3 < 0$ if and only if

$$\begin{bmatrix} \mathcal{S}_1 & \mathcal{S}_3^{\mathrm{T}} \\ \mathcal{S}_3 & -\mathcal{S}_2 \end{bmatrix} < 0, \quad \text{or} \quad \begin{bmatrix} -\mathcal{S}_2 & \mathcal{S}_3 \\ \mathcal{S}_3^{\mathrm{T}} & \mathcal{S}_1 \end{bmatrix} < 0. \tag{18}$$

Lemma 2: (S-procedure [2]) Let $\kappa_0(\cdot)$,$\kappa_1(\cdot)$,…,$\kappa_s(\cdot)$ be quadratic functions of the variable $\zeta \in \mathbb{R}^n$: $\kappa_j(\zeta) \triangleq \zeta^{\mathrm{T}} T_j \zeta$ $(j = 0, 1, \ldots, s)$, where $T_j^{\mathrm{T}} = T_j$. If there exist $\tau_1 \geq 0$, …, $\tau_s \geq 0$ such that $T_0 - \sum_{j=1}^{s} \tau_j T_j \leq 0$, then the following is true:

$$\kappa_1(\zeta) \leq 0, \ldots, \kappa_s(\zeta) \leq 0 \Longrightarrow \kappa_0(\zeta) \leq 0. \tag{19}$$

From the system (14) and the filter (15), for sensing node i $(i = 1, 2, \ldots, N)$, the one-step ahead estimation error is calculated by

$$\begin{aligned} & x_{k+1} - \hat{x}_{i,k+1} \\ &= \left(A_k + \sum_{l=1}^{q} \alpha_{l,k} A_{l,k}\right) x_k - B_k\underline{\Gamma}u_k + B_k\underline{\Gamma}\delta_k + B_k\varphi(\vec{u}_k) \\ &\quad + D_k w_k - G_{i,k}\hat{x}_{i,k} - \sum_{j \in \mathcal{N}_i} \theta_{ij} K_{ij,k}(I - \Xi_i) C_{i,k} x_k \\ &\quad - \sum_{j \in \mathcal{N}_i} \theta_{ij} K_{ij,k}(I - \Xi_i) E_{i,k} v_k - \sum_{j \in \mathcal{N}_i} \theta_{ij} K_{ij,k}\Xi_i\vartheta_{i,k} \\ &\quad - \sum_{j \in \mathcal{N}_i} \theta_{ij} K_{ij,k}\psi_i(\vec{y}_{i,k}) + \sum_{j \in \mathcal{N}_i} \theta_{ij} K_{ij,k} C_{i,k}\hat{x}_{i,k}. \end{aligned} \tag{20}$$

For simplicity of the further development, we denote

$$\mathcal{T}_{\lambda,i} \triangleq [\ \underbrace{0\ \cdots\ 0}_{i-1}\ I_\lambda\ \underbrace{0\ \cdots\ 0}_{N-i}\ ],$$
$$\Omega_{\lambda,i} \triangleq \mathcal{T}_{\lambda,i}^{\mathrm{T}}\mathcal{T}_{\lambda,i}(I_N \otimes I_\lambda),\ \lambda = \{n,q,p\},\ i = 1,\ldots,N.$$

Denoting $\tilde{x}_{i,k} \triangleq x_k - \hat{x}_{i,k}$, we can rewrite (20) into a compact form as follows:

$$\begin{aligned}\tilde{x}_{k+1} =&(\mathcal{A}_k + \bar{\alpha}_k\bar{\mathcal{A}}_k)\zeta_k - \mathcal{B}_k\mu_k + \mathcal{B}_k\tilde{\delta}_k + \bar{\mathcal{B}}_k\tilde{\varphi}_k\\ &+ \mathcal{D}_k\tilde{w}_k - \mathcal{G}_k\hat{x}_k - \mathcal{K}_k(I-\Xi)\mathcal{C}_k\zeta_k\\ &- \mathcal{K}_k(I-\Xi)\mathcal{E}_k\tilde{v}_k - \mathcal{K}_k\Xi\vartheta_k\\ &- \mathcal{K}_k\psi_k + \mathcal{K}_k\mathcal{C}_k\hat{x}_k \end{aligned} \tag{21}$$

where

$$\begin{aligned}&\tilde{x}_k \triangleq \mathrm{col}_N\{\tilde{x}_{i,k}\},\ \zeta_k \triangleq \mathrm{col}_N\{x_k\},\ \hat{x}_k \triangleq \mathrm{col}_N\{\hat{x}_{i,k}\},\\ &\mu_k \triangleq \mathrm{col}_N\{u_k\},\ \tilde{\delta}_k \triangleq \mathrm{col}_N\{\delta_k\},\ \tilde{\varphi}_k \triangleq \mathrm{col}_N\{\varphi(\vec{u}_k)\},\\ &\tilde{w}_k \triangleq \mathrm{col}_N\{w_k\},\ \tilde{v}_k \triangleq \mathrm{col}_N\{v_k\},\ \vartheta_k \triangleq \mathrm{col}_N\{\vartheta_{i,k}\},\\ &\psi_k \triangleq \mathrm{col}_N\{\psi_i(\vec{y}_{i,k})\},\ \mathcal{A}_k \triangleq \mathrm{diag}_N\{A_k\},\\ &\tilde{\mathcal{A}}_k \triangleq \mathrm{diag}_N\Big\{\sum\nolimits_{l=1}^{q} A_{l,k}\Big\},\ \alpha_k \triangleq [\ \alpha_{1,k}I\ \ \cdots\ \ \alpha_{q,k}I\ ],\\ &\bar{A}_k \triangleq [\ A_{1,k}^{\mathrm{T}}\ \ A_{2,k}^{\mathrm{T}}\ \ \cdots\ \ A_{q,k}^{\mathrm{T}}\ ]^{\mathrm{T}},\ \bar{\alpha}_k \triangleq \mathrm{diag}_N\{\alpha_k\},\\ &\bar{\mathcal{A}}_k \triangleq \mathrm{diag}_N\{\bar{A}_k\},\ \mathcal{B}_k \triangleq \mathrm{diag}_N\{B_k\underline{\Gamma}\},\ \bar{\mathcal{B}}_k \triangleq \mathrm{diag}_N\{B_k\},\\ &\mathcal{C}_k \triangleq \mathrm{diag}_N\{C_{i,k}\},\ \mathcal{D}_k \triangleq \mathrm{diag}_N\{D_k\},\ \mathcal{E}_k \triangleq \mathrm{diag}_N\{E_{i,k}\},\\ &\mathcal{G}_k \triangleq \mathrm{diag}_N\{G_{i,k}\},\ \Xi \triangleq \mathrm{diag}_N\{\Xi_i\},\ \mathcal{K}_k \triangleq [\theta_{ij}K_{ij,k}]_{N\times N}.\end{aligned}$$

Noticing that when $j \notin \mathcal{N}_i$, $\theta_{ij} = 0$, we know that $\mathcal{K}_k$ is a sparse matrix which can be described by

$$\mathcal{K}_k \in \mathscr{Q}_{n\times m} \tag{22}$$

where $\mathscr{Q}_{n\times m} \triangleq \{\mathcal{Q} = [Q_{ij}] \in \mathbb{R}^{nN\times mN} \big| Q_{ij} \in \mathbb{R}^{n\times m}, Q_{ij} = 0 \text{ if } j \notin \mathcal{N}_i\}$.

The following theorem presents a sufficient condition for the existence of the desired distributed filter by RLMI approach.

Theorem 1: For the system (1)–(2) subject to the deception attacks (6), let the network topology $\mathscr{G}$ and the prespecified sequence of variance constraints $\{\Phi_k\}_{k\geq 0}$ be given. The design objective (17) is achieved if there exist sequences of real-valued matrices $\{\mathcal{G}_k\}_{k\geq 0}$ and $\{\mathcal{K}_k\}_{k\geq 0}$ ($\mathcal{K}_k \in \mathscr{Q}_{n\times m}$), sequences of positive definite matrices $\{\tilde{S}_k\}_{k\geq 0}$ and $\{\tilde{R}_{i,k}\}_{k\geq 0}$, sequences of non-negative scalars $\{\tau_{1,k}\}_{k\geq 0}$, $\{\tau_{2,k}\}_{k\geq 0}$, $\{\tau_{3,k}\}_{k\geq 0}$, $\{\tau_{4,k}\}_{k\geq 0}$, $\{\epsilon_{i,k}\}_{k\geq 0}$, $\{\varrho_{i,k}\}_{k\geq 0}$ and $\{\rho_{i,k}\}_{k\geq 0}$ $(i = 1,2,\ldots,N)$ satisfying the following N RLMIs:

$$\begin{bmatrix} -\Delta_k & * & * \\ \mathcal{T}_{n,i}\bar{\Lambda}_k & -\Phi_{k+1} & * \\ \mathcal{T}_{n,i}\hat{\Lambda}_k & 0 & -\Phi_{k+1}\end{bmatrix} \leq 0, \qquad i = 1,2,\ldots,N \tag{23}$$

where

$$\begin{aligned}\Delta_k \triangleq\ & \tau_{4,k}\Pi_k + \sum_{i=1}^{N}\rho_{i,k}\Upsilon_{i,k}\\ &+ \mathrm{diag}\Big\{1 - \sum_{i=1}^{N}(\epsilon_{i,k}+\varrho_{i,k}) - \tau_{1,k} - \tau_{2,k} - \tau_{3,k},\\ &\sum_{i=1}^{N}\epsilon_{i,k}\mathcal{T}_{q,i}^{\mathrm{T}}\mathcal{T}_{q,i}\tilde{S}_k, \sum_{i=1}^{N}\mathcal{T}_{p,i}^{\mathrm{T}}\tilde{R}_{i,k}\mathcal{T}_{p,i},\\ &\tau_{2,k}W_k^{-1}, \tau_{3,k}V_k^{-1}, 0, 0\Big\},\end{aligned} \tag{24}$$

$$\bar{\Pi}_k \triangleq \begin{bmatrix} u_k^{\mathrm{T}}\tilde{\Gamma}\\ 0\\ -\tilde{\Gamma}\\ 0\\ 0\\ 0\\ 2I_m\\ 0\end{bmatrix},\ \bar{\Upsilon}_{i,k} \triangleq \begin{bmatrix} \hat{x}_k^{\mathrm{T}}\Omega_{n,i}\mathrm{diag}_N\{C_{i,k}^{\mathrm{T}}\tilde{\Xi}_i\}\Omega_{p,i}\\ \Omega_{q,i}\mathrm{diag}_N\{P_k^{\mathrm{T}}C_{i,k}^{\mathrm{T}}\tilde{\Xi}_i\}\Omega_{p,i}\\ 0\\ -\Omega_{p,i}\mathrm{diag}_N\{\tilde{\Xi}_i\}\Omega_{p,i}\\ 0\\ \mathcal{T}_{p,i}\mathrm{diag}_N\{E_{i,k}^{\mathrm{T}}\tilde{\Xi}_i\}\Omega_{p,i}\\ 0\\ 2\Omega_{p,i}I_{pN}\Omega_{p,i}\end{bmatrix},$$

$$\Pi_k \triangleq \frac{1}{2}[\ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \bar{\Pi}_k\ \ 0\ ], \tag{25}$$

$$\Upsilon_{i,k} \triangleq \frac{1}{2}[\ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \bar{\Upsilon}_{i,k}\ ], \tag{26}$$

$$\mathscr{I}_m \triangleq \mathbf{1}_N \otimes I_m,\quad \mathscr{I}_\omega \triangleq \mathbf{1}_N \otimes I_\omega,\quad \mathscr{I}_\nu \triangleq \mathbf{1}_N \otimes I_\nu, \tag{27}$$

$$\begin{aligned}&\bar{\Lambda}_{11} \triangleq \Big(\mathcal{A}_k - \mathcal{G}_k + \mathcal{K}_k\Xi\mathcal{C}_k\Big)\hat{x}_k - \mathcal{B}_k\mu_k,\\ &\bar{\Lambda}_{12} \triangleq \mathcal{A}_k\mathcal{P}_k - \mathcal{K}_k(I-\Xi)\mathcal{C}_k\mathcal{P}_k,\\ &\bar{\Lambda}_k \triangleq [\ \bar{\Lambda}_{11}\ \ \bar{\Lambda}_{12}\ \ \mathcal{B}_k\mathscr{I}_m\ \ -\mathcal{K}_k\Xi\ \ \mathcal{D}_k\mathscr{I}_\omega\\ &\qquad -\mathcal{K}_k(I-\Xi)\mathcal{E}_k\mathscr{I}_\nu\ \ \bar{\mathcal{B}}_k\mathscr{I}_m\ \ -\mathcal{K}_k\ ],\end{aligned} \tag{28}$$

$$\hat{\Lambda}_k \triangleq [\ \tilde{\mathcal{A}}_k\hat{x}_k\ \ \tilde{\mathcal{A}}_k\mathcal{P}_k\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ ] \tag{29}$$

with P_k being a factorization of Φ_k (i.e., $\Phi_k = P_kP_k^{\mathrm{T}}$) and $\mathcal{P}_k \triangleq \mathrm{diag}_N\{P_k\}$. Moreover, the parameters S_k and $R_{i,k}$ can be computed by $S_k = \tau_{1,k}\tilde{S}_k^{-1}$ and $R_{i,k} = \varrho_{i,k}\tilde{R}_{i,k}^{-1}$.

Proof: We prove Theorem 1 by induction which can be divided into two steps, namely, the initial step and the inductive step.

Initial step. For $k = 0$, it can be known directly from Assumption 2 that the initial value of the state x_0 and its estimates $\hat{x}_{i,0}$ $(i = 1,2,\ldots,N)$ satisfy

$$\mathbb{E}\{(x_0 - \hat{x}_{i,0})(x_0 - \hat{x}_{i,0})^{\mathrm{T}}\} \leq \Phi_0. \tag{30}$$

Inductive step. Given that at the time instant $k > 0$, the following is true:

$$\mathbb{E}\{(x_k - \hat{x}_{i,k})(x_k - \hat{x}_{i,k})^{\mathrm{T}}\} \leq \Phi_k, \tag{31}$$

then we aim to, with the condition given in the theorem, demonstrate that the following set of inequalities holds for all i:

$$\mathbb{E}\{(x_{k+1} - \hat{x}_{i,k+1})(x_{k+1} - \hat{x}_{i,k+1})^{\mathrm{T}}\} \leq \Phi_{k+1}. \tag{32}$$

Since the inequality (31) is true, it follows from [9] that there exists a sequence of vectors $z_{i,k} \in \mathbb{R}^q$ (with $\mathbb{E}\{z_{i,k}^{\mathrm{T}}z_{i,k}\} \leq 1$) satisfying $x_k = \hat{x}_{i,k} + P_kz_{i,k}$ where $P_k \in \mathbb{R}^{n\times q}$ is a factorization of $\Phi_k = P_kP_k^{\mathrm{T}}$. Denoting $z_k \triangleq \mathrm{col}_N\{z_{i,k}\}$ and noticing $\mathcal{P}_k = \mathrm{diag}_N\{P_k\}$, we can further acquire that

$$\zeta_k = \hat{x}_k + \mathcal{P}_kz_k. \tag{33}$$

Now, substituting (33) into (21) yields

$$\tilde{x}_{k+1} = \big(\mathcal{A}_k + \bar{\alpha}_k\bar{\mathcal{A}}_k - \mathcal{G}_k - \mathcal{K}_k(I-\Xi)\mathcal{C}_k + \mathcal{K}_k\mathcal{C}_k\big)\hat{x}_k$$

$$
\begin{aligned}
&- \mathcal{B}_k\mu_k + \big((\mathcal{A}_k + \bar{\alpha}_k\bar{\mathcal{A}}_k)\mathcal{P}_k - \mathcal{K}_k(I-\Xi)\mathcal{C}_k\mathcal{P}_k\big)z_k \\
&+ \mathcal{B}_k\mathscr{I}_m\delta_k + \bar{\mathcal{B}}_k\mathscr{I}_m\varphi_k + \mathcal{D}_k\mathscr{I}_\omega w_k \\
&- \mathcal{K}_k(I-\Xi)\mathcal{E}_k\mathscr{I}_\nu v_k - \mathcal{K}_k\Xi\vartheta_k - \mathcal{K}_k\psi_k. \qquad (34)
\end{aligned}
$$

We now define a vector as follows:

$$
\beta_k \triangleq \left[\begin{array}{ccccccccc} 1 & z_k^{\mathrm{T}} & \delta_k^{\mathrm{T}} & \vartheta_k^{\mathrm{T}} & w_k^{\mathrm{T}} & v_k^{\mathrm{T}} & \varphi_k^{\mathrm{T}} & \psi_k^{\mathrm{T}} \end{array}\right]^{\mathrm{T}}. \qquad (35)
$$

Then, the one-step ahead estimation error $\tilde{x}_{k+1}$ in (34) is expressed by

$$
\tilde{x}_{k+1} = \Lambda_k\beta_k \qquad (36)
$$

where

$$
\begin{aligned}
\Lambda_{11} &\triangleq (\mathcal{A}_k + \bar{\alpha}_k\bar{\mathcal{A}}_k - \mathcal{G}_k + \mathcal{K}_k\Xi\mathcal{C}_k + \mathcal{G}_k)\hat{x}_k - \mathcal{B}_k\mu_k, \\
\Lambda_{12} &\triangleq (\mathcal{A}_k + \bar{\alpha}_k\bar{\mathcal{A}}_k)\mathcal{P}_k - \mathcal{K}_k(I-\Xi)\mathcal{C}_k\mathcal{P}_k + \mathcal{G}_k\mathcal{P}_k, \\
\Lambda_k &\triangleq \left[\begin{array}{cccc} \Lambda_{11} & \Lambda_{12} & \mathcal{B}_k\mathscr{I}_m & -\mathcal{K}_k\Xi \\ & & \mathcal{D}_k\mathscr{I}_\omega & -\mathcal{K}_k(I-\Xi)\mathcal{E}_k\mathscr{I}_\nu \end{array}\right. \\
&\qquad \left.\begin{array}{cc} \bar{\mathcal{B}}_k\mathscr{I}_m & -\mathcal{K}_k \end{array}\right]. \qquad (37)
\end{aligned}
$$

Next, we decompose the matrix Λ_k into a deterministic part $\bar{\Lambda}_k$ defined in (28) and a stochastic part $\tilde{\Lambda}_k$ which contains the random variable $\bar{\alpha}_k$ as follows:

$$
\Lambda_k = \bar{\Lambda}_k + \tilde{\Lambda}_k \qquad (38)
$$

where

$$
\tilde{\Lambda}_k \triangleq \left[\begin{array}{cccccccc} \bar{\alpha}_k\bar{\mathcal{A}}_k\hat{x}_k & \bar{\alpha}_k\bar{\mathcal{A}}_k\mathcal{P}_k & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right].
$$

Then, by taking into consideration the statistical properties of the random variable $\bar{\alpha}_k$, we have

$$
\begin{aligned}
&\mathbb{E}\{\Lambda_k^{\mathrm{T}}\mathcal{T}_{n,i}^{\mathrm{T}}\Phi_{k+1}^{-1}\mathcal{T}_{n,i}\Lambda_k\} \\
=&\bar{\Lambda}_k^{\mathrm{T}}\mathcal{T}_{n,i}^{\mathrm{T}}\Phi_{k+1}^{-1}\mathcal{T}_{n,i}\bar{\Lambda}_k + \hat{\Lambda}_k^{\mathrm{T}}\mathcal{T}_{n,i}^{\mathrm{T}}\Phi_{k+1}^{-1}\mathcal{T}_{n,i}\hat{\Lambda}_k. \qquad (39)
\end{aligned}
$$

In the following, we shall proceed to deal with the constraints (13) imposed on the attack signals $\vec{u}_k$ and $\vec{y}_{i,k}$ $(i = 1, 2, \ldots, N)$. It can be known from (13) that the nonlinear vector-valued functions $\varphi(\vec{u}_k)$ and $\psi_i(\vec{y}_{i,k})$ $(i = 1, 2, \ldots, N)$ are belonging to the sectors $[0, \tilde{\Gamma}]$ and $[0, \tilde{\Xi}_i]$ $(i = 1, 2, \ldots, N)$, respectively. As such, we can perform the following derivations:

$$
\varphi^{\mathrm{T}}(\vec{u}_k)\big(\varphi(\vec{u}_k) - \tilde{\Gamma}\vec{u}_k\big) \le 0 \Longleftrightarrow \beta_k^{\mathrm{T}}\Pi_k\beta_k \le 0 \qquad (40)
$$

where Π_k is defined in (25).

Likewise, we have

$$
\beta_k^{\mathrm{T}}\Upsilon_{i,k}\beta_k \le 0 \qquad (41)
$$

where $\Upsilon_{i,k}$ is defined in (26).

For the brevity of presentation, we denote

$$
\begin{aligned}
N_{i,k} &\triangleq \mathrm{diag}\left\{-1, \mathcal{T}_{q,i}^{\mathrm{T}}\mathcal{T}_{q,i}, 0, 0, 0, 0, 0, 0\right\}, \\
M_{1,k} &\triangleq \mathrm{diag}\left\{-1, 0, S_k^{-1}, 0, 0, 0, 0, 0\right\}, \\
J_{i,k} &\triangleq \mathrm{diag}\left\{-1, 0, 0, \mathcal{T}_{p,i}^{\mathrm{T}}R_{i,k}^{-1}\mathcal{T}_{p,i}, 0, 0, 0, 0\right\}, \\
M_{2,k} &\triangleq \mathrm{diag}\left\{-1, 0, 0, 0, W_k^{-1}, 0, 0, 0\right\}, \\
M_{3,k} &\triangleq \mathrm{diag}\left\{-1, 0, 0, 0, 0, V_k^{-1}, 0, 0\right\}.
\end{aligned}
$$

According to Definition 2, it is not difficult to reformulate the constraints $\mathbb{E}\{z_{i,k}^{\mathrm{T}}z_{i,k}\} \le 1$, $\delta_k \in \mathscr{E}(0, S_k, m)$, $\vartheta_{i,k} \in \mathscr{E}(0, R_{i,k}, p)$, $w_k \in \mathscr{E}(0, W_k, \omega)$ and $v_k \in \mathscr{E}(0, V_k, \nu)$ in terms of the variable β_k as follows:

$$
\begin{aligned}
&\beta_k^{\mathrm{T}}N_{i,k}\beta_k \le 0,\ \beta_k^{\mathrm{T}}M_{1,k}\beta_k \le 0,\ \beta_k^{\mathrm{T}}J_{i,k}\beta_k \le 0, \\
&\beta_k^{\mathrm{T}}M_{2,k}\beta_k \le 0,\ \beta_k^{\mathrm{T}}M_{3,k}\beta_k \le 0.
\end{aligned} \qquad (42)
$$

By applying the Schur Complement Lemma (Lemma 1) to the set of RLMIs (23) and noting that $S_k = \tau_{1,k}\tilde{S}_k^{-1}$ and $R_{i,k} = \varrho_{i,k}\tilde{R}_{i,k}^{-1}$, we obtain

$$
\bar{\Lambda}_k^{\mathrm{T}}\mathcal{T}_{n,i}^{\mathrm{T}}\Phi_{k+1}^{-1}\mathcal{T}_{n,i}\bar{\Lambda}_k + \hat{\Lambda}_k^{\mathrm{T}}\mathcal{T}_{n,i}^{\mathrm{T}}\Phi_{k+1}^{-1}\mathcal{T}_{n,i}\hat{\Lambda}_k - \Delta_k \le 0 \qquad (43)
$$

which, by (24), is equivalent to

$$
\begin{aligned}
&\bar{\Lambda}_k^{\mathrm{T}}\mathcal{T}_{n,i}^{\mathrm{T}}\Phi_{k+1}^{-1}\mathcal{T}_{n,i}\bar{\Lambda}_k + \hat{\Lambda}_k^{\mathrm{T}}\mathcal{T}_{n,i}^{\mathrm{T}}\Phi_{k+1}^{-1}\mathcal{T}_{n,i}\hat{\Lambda}_k \\
&-\mathrm{diag}\{1,0,0,0,0,0,0,0\} \\
&-\tau_{1,k}M_{1,k} - \tau_{2,k}M_{2,k} - \tau_{3,k}M_{3,k} - \tau_{4,k}\Pi_k \\
&-\sum_{i=1}^{N}\epsilon_{i,k}N_{i,k} - \sum_{i=1}^{N}\rho_{i,k}\Upsilon_{i,k} - \sum_{i=1}^{N}\varrho_{i,k}J_{i,k} \le 0. \qquad (44)
\end{aligned}
$$

By resorting to Lemma 2 and on account of (40), (41) and (42), we immediately arrive at

$$
\beta_k^{\mathrm{T}}\big(\bar{\Lambda}_k^{\mathrm{T}}\mathcal{T}_{n,i}^{\mathrm{T}}\Phi_{k+1}^{-1}\mathcal{T}_{n,i}\bar{\Lambda}_k + \hat{\Lambda}_k^{\mathrm{T}}\mathcal{T}_{n,i}^{\mathrm{T}}\Phi_{k+1}^{-1}\mathcal{T}_{n,i}\hat{\Lambda}_k\big)\beta_k \le 1, \qquad (45)
$$

which, by means of (39), further indicates that

$$
\mathbb{E}\{\beta_k^{\mathrm{T}}\Lambda_k^{\mathrm{T}}\mathcal{T}_{n,i}^{\mathrm{T}}\Phi_{k+1}^{-1}\mathcal{T}_{n,i}\Lambda_k\beta_k\} \le 1, \qquad (46)
$$

or equivalently,

$$
\mathbb{E}\{\tilde{x}_{i,k+1}^{\mathrm{T}}\Phi_{k+1}^{-1}\tilde{x}_{i,k+1}\} \le 1. \qquad (47)
$$

By using Schur Complement Lemma again, the inequality (47) holds if and only if

$$
\mathbb{E}\{\tilde{x}_{i,k+1}\tilde{x}_{i,k+1}^{\mathrm{T}}\} \le \Phi_{k+1}, \qquad (48)
$$

which accomplishes the induction. Accordingly, we conclude that the design objective (17) is achieved subject to the fixed communication topology $\mathscr{G}$ and variance constraints $\{\Phi_k\}_{k\ge 0}$. The desired sequences of filtering parameters $\{\mathcal{G}_k\}_{k\ge 0}$ and $\{\mathcal{K}_k\}_{k\ge 0}$ can be obtained by solving the set of RLMIs (23) iteratively. The proof is now complete. ■

In the following stage, an optimization problem is formulated with the purpose to determine the filtering gains ensuring the locally optimal filtering performance by minimizing Φ_k in the sense of matrix trace at each time instant.

Corollary 1: For the system (1)–(2) subject to the deception attacks (6), let the network topology $\mathscr{G}$ be given. A sequence of minimized $\{\Phi_k\}_{k\ge 1}$ can be guaranteed (in the sense of matrix trace) if there exist sequences of real-valued matrices $\{\mathcal{G}_k\}_{k\ge 0}$ and $\{\mathcal{K}_k\}_{k\ge 0}$ $(\mathcal{K}_k \in \mathscr{Q}_{n\times m})$, sequences of positive definite matrices $\{\tilde{S}_k\}_{k\ge 0}$ and $\{\tilde{R}_{i,k}\}_{k\ge 0}$, sequences of nonnegative scalars $\{\tau_{1,k}\}_{k\ge 0}$, $\{\tau_{2,k}\}_{k\ge 0}$, $\{\tau_{3,k}\}_{k\ge 0}$, $\{\tau_{4,k}\}_{k\ge 0}$, $\{\epsilon_{i,k}\}_{k\ge 0}$, $\{\varrho_{i,k}\}_{k\ge 0}$ and $\{\rho_{i,k}\}_{k\ge 0}$ $(i = 1, 2, \ldots, N)$ solving the following optimization problem:

$$
\min_{\Phi_{k+1},\mathcal{G}_k,\mathcal{K}_k,\tilde{S}_k,\tilde{R}_{i,k},\tau_{1,k},\tau_{2,k},\tau_{3,k},\tau_{4,k},\epsilon_{i,k},\varrho_{i,k},\rho_{i,k}} \mathrm{tr}[\Phi_{k+1}] \qquad (49)
$$

$$
\text{subject to} \quad \left[\begin{array}{ccc} -\Delta_k & * & * \\ \mathcal{T}_{n,i}\bar{\Lambda}_k & -\Phi_{k+1} & * \\ \mathcal{T}_{n,i}\hat{\Lambda}_k & 0 & -\Phi_{k+1} \end{array}\right] \le 0.
$$

Remark 4: So far, the addressed variance-constrained distributed filtering problem has been discussed for the time-varying systems with multiplicative noises, unknown but bounded disturbances and deception attacks. The existence condition of the desired distributed filter has been established by resorting to the recursive linear matrix inequality approach. The filter gains can be determined via solving a set of RLMIs iteratively. The optimization problem for the locally optimal filtering performance has also been solved by minimizing the constraints imposed on the estimation error variance in the sense of matrix trace. One of our future research topics is the variance-constrained distributed filtering subject to other frequently seen network-induced complexities such as transmission delay [4], [20], [37], multiple missing measurements [15], [22] and quantization effects [19], [35], [36]. It is also our interest to apply the proposed technique to deal with the consensus control problems for stochastic multi-agent systems studied in [17], where the open communication network may be also under attacks.

IV. Numerical Example

In this section, an illustrative example is presented to illustrate the effectiveness of the proposed algorithm. We consider a target tracking system whose dynamic model is given as follows:

$$x_{k+1} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} + Dw_k \tag{50}$$

where T is the sampling period and the state $x_k = [s_k \ \dot{s}_k]^{\mathrm{T}}$ with s_k and $\dot{s}_k$ being the position and velocity, respectively. It is worth mentioning in (50), the system parameters are time-invariant and the state is only subject to the external additive noise w_k. However, in the real-world application, because of the changeable circumstance, these parameters are usually time-varying. Moreover, the system may contain, apart from the additive noises, certain multiplicative disturbances that have significant impact on the performance [11]. Taking these into account, we propose the model of system (1)–(2) with following parameters that are of more practical significance:

$$A_k = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \ T = 0.1,$$
$$A_{1,k} = \begin{bmatrix} 0.08 + 0.1\sin(3k) & -0.05 \\ 0.01 & -0.01 + 0.02\cos(k) \end{bmatrix},$$
$$A_{2,k} = \begin{bmatrix} 0.06 + 0.01\sin(10k) & 0.02 \\ -0.01 & -0.07 + 0.02\cos(k) \end{bmatrix},$$
$$C_{1,k} = \begin{bmatrix} 1 & 0 \end{bmatrix},$$
$$C_{2,k} = \begin{bmatrix} 0.9 + \sin(k) & 0 \end{bmatrix},$$
$$C_{3,k} = \begin{bmatrix} 0.75 + 0.5\sin(k) & 0.1 \end{bmatrix},$$
$$D_k = \begin{bmatrix} 0.1 + 0.05\cos(3k) \\ 0.2 + 0.04\exp\{-k\} \end{bmatrix},$$
$$E_{1,k} = 0.2 + 0.05\cos(3k),$$
$$E_{2,k} = 0.2 + 0.15\sin(2k),$$
$$E_{3,k} = 0.15 + 0.05\sin(2k).$$

Suppose the unknown but bounded disturbances are $w_k = 0.2\sin(5k)$ and $v_k = 0.5\cos(2k)$ and set $W_k = 0.04$, $V_k = 0.25$. Then it can be easily checked that w_k and v_k belong to the ellipsoid sets defined in (5).

Suppose that there are three sensor nodes connected according to graph $\mathscr{G}$. The associated adjacency matrix $\mathscr{L}$ is selected as follows:

$$\mathscr{L} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Set the initial values as follows:

$$x_0 = \begin{bmatrix} 5 \\ 3 \end{bmatrix}, \ \hat{x}_{1,0} = \begin{bmatrix} 2.8 \\ 1.6 \end{bmatrix}, \ \hat{x}_{2,0} = \begin{bmatrix} 3.0 \\ 2.0 \end{bmatrix},$$
$$\hat{x}_{3,0} = \begin{bmatrix} 3.2 \\ 1.2 \end{bmatrix}, \ \Phi_0 = \begin{bmatrix} 15 & 0.1 \\ 0.1 & 15 \end{bmatrix}.$$

Then it can be easily checked that (16) is satisfied.

Suppose that the that the constraints imposed on the attack signals are characterized by $\underline{\Gamma} = 0.8$, $\bar{\Gamma} = 1.2$, $\underline{\Xi}_1 = 0.8$, $\bar{\Xi}_1 = 1.2$, $\underline{\Xi}_2 = 0.9$, $\bar{\Xi}_2 = 1.1$, $\underline{\Xi}_3 = 0.7$ and $\bar{\Xi}_3 = 1.3$.

In this section, we proceed to utilize the algorithm proposed in Corollary 1 to solve the optimization problem (49). By using Matlab software, the simulation is carried out and the results are shown in Figs. 2–7. Specifically, Figs. 2–3 depict the trajectories of the individual entries of the system state x_k (i.e., $x_k^{(1)}$ and $x_k^{(2)}$) and their estimates at each sensing node. Figs. 4–5 present the trajectories of the one-step-ahead estimation errors (i.e., $\tilde{x}_{i,k}$) of all the sensing nodes. The estimation error variances of all the sensing nodes are proposed in Fig. 6–7, which indicate that the proposed algorithm performs quite well.

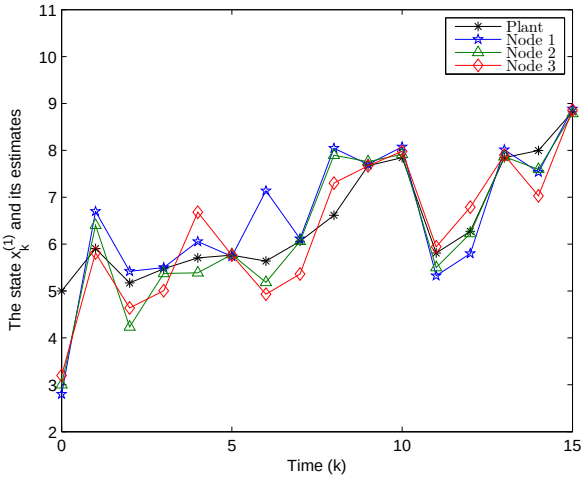

Fig. 2. The state $x_k^{(1)}$ and its estimate.

V. Conclusion

The variance-constrained distributed filtering problem has been studied for a class of discrete time-varying systems with multiplicative noises, unknown but bounded disturbances and deception attacks over sensor networks. A novel deception attack model has been proposed, where the attack signals are injected by the adversary to both control and measurement

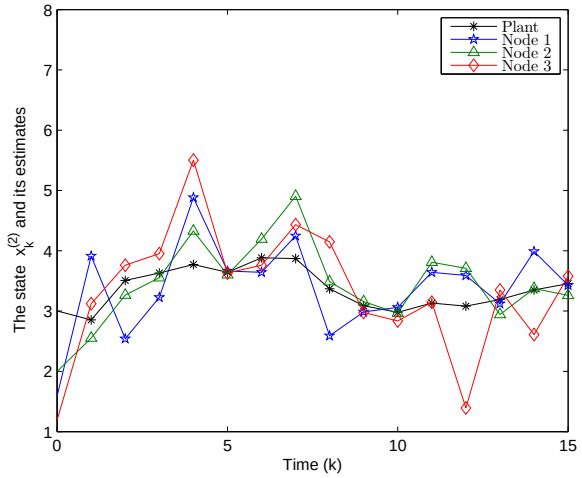

Fig. 3. The state $x_k^{(2)}$ and its estimate.

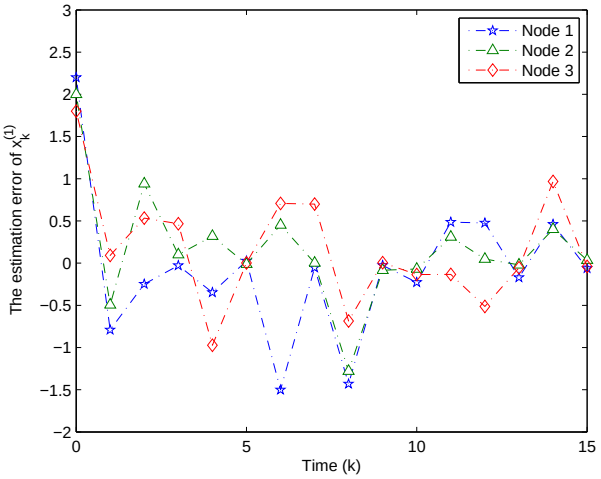

Fig. 4. The estimation error of $x_k^{(1)}$.

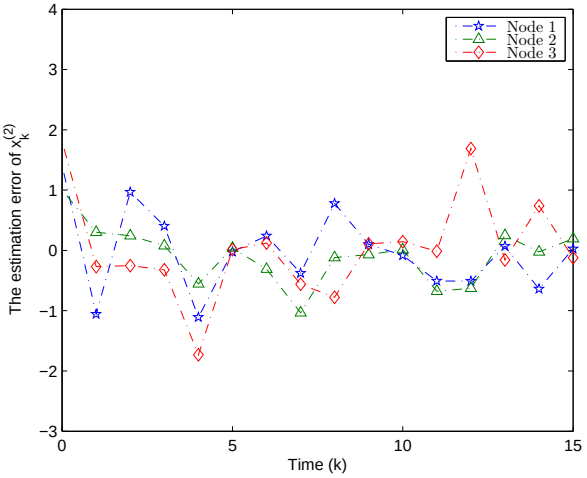

Fig. 5. The estimation error of $x_k^{(2)}$.

data during the transmission via the communication networks. A sufficient condition has been established for the existence of the required filter satisfying the estimation error variance constraints by means of the RLMI approach. An optimization problem has been presented to seek the filter parameters with the guarantee of the locally minimal estimation error variance at each time instant. Finally, an illustrative example has been used to show the effectiveness and applicability of the proposed algorithm. It is worth mentioning that our proposed filtering scheme is actually a robust technique against deception attacks. In practical engineering, certain attack detectors such as the widely used χ^2 detector are usually implemented. However, χ^2 detector is only effective to distinguish deception signals obeying Gaussian distribution, which is therefore cannot be applied to prevent the unknown but bounded deception signals considered in this paper. It is of practical significance to exploit novel mechanisms that are capable of detecting other types of attack signals aside from Gaussian noises.

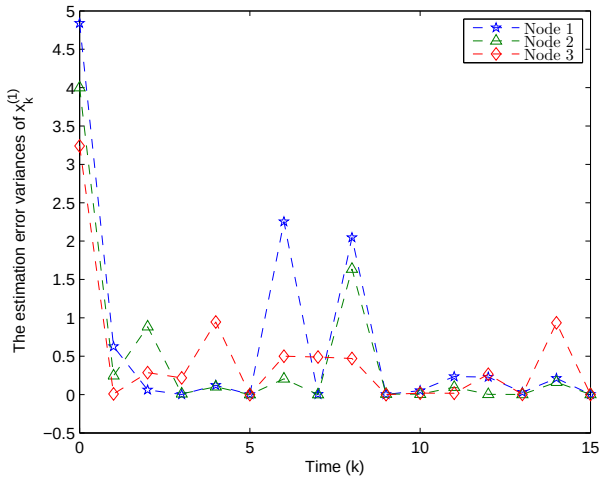

Fig. 6. The variance of $\tilde{x}_k^{(1)}$.

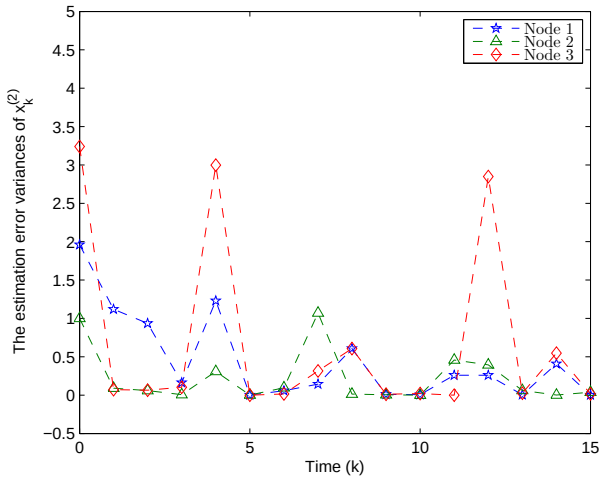

Fig. 7. The variance of $\tilde{x}_k^{(2)}$.

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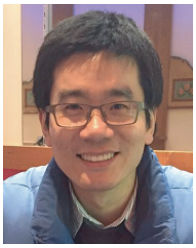

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